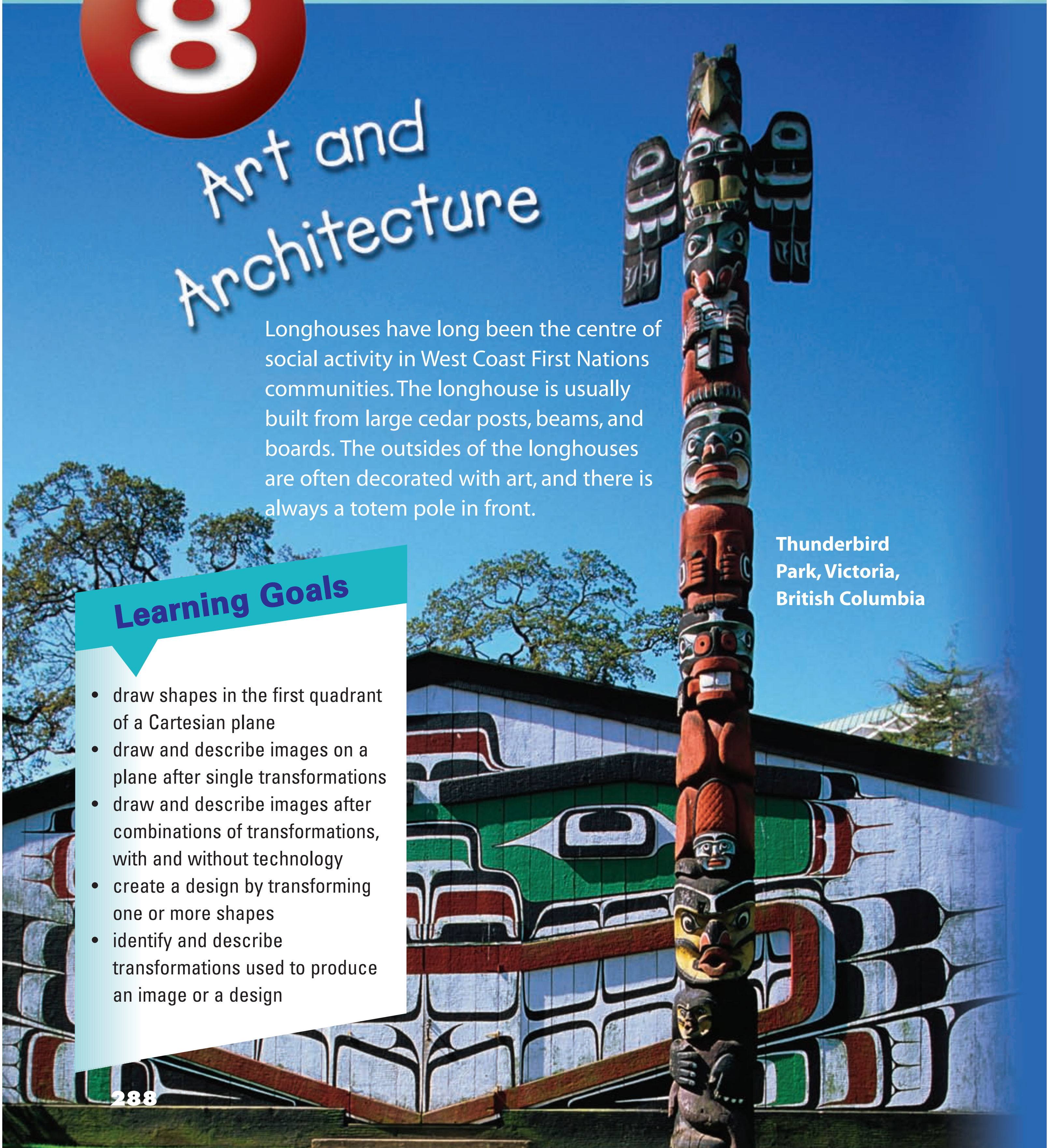
UNIT 8

Transformations

Art and Architecture

Longhouses have long been the centre of social activity in West Coast First Nations communities. The longhouse is usually built from large cedar posts, beams, and boards. The outsides of the longhouses are often decorated with art, and there is always a totem pole in front.



Thunderbird Park, Victoria, British Columbia

Learning Goals

- draw shapes in the first quadrant of a Cartesian plane
- draw and describe images on a plane after single transformations
- draw and describe images after combinations of transformations, with and without technology
- create a design by transforming one or more shapes
- identify and describe transformations used to produce an image or a design

K'san Village, Hazelton, British Columbia

Key Words

successive translations

successive rotations

successive reflections

In 1993, the University of British Columbia opened The First Nations Longhouse. It is a meeting place and library for First Nations students.

The construction was overseen by First Nations elders and it reflects the architectural traditions of the Northwest Coast.

First Nations Longhouse, University of British Columbia

- Describe the photographs you see.
- Which transformations are shown in the photographs?
- How did you identify the transformations?

LESSON 1

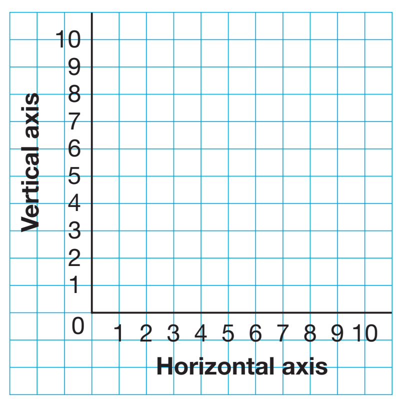
Drawing Shapes on a Coordinate Grid

Here is a plan for an amusement park drawn on a coordinate grid.

What are the coordinates of the water ride? The swinging ship?

Explore

You will need 1-cm grid paper and a ruler.
Copy this grid.



Take turns.
Draw a shape on the grid.
Do not show your partner the shape.
Describe the shape you drew and its position to your partner.
Your partner draws the shape as you describe it.
Compare shapes. What do you notice?

Show *and* Share

Talk with another pair of classmates.
Trade ideas for describing the position of a shape on a grid.
Did your shapes match exactly?
If not, how could you have improved your description?
How can you tell that two shapes match exactly?

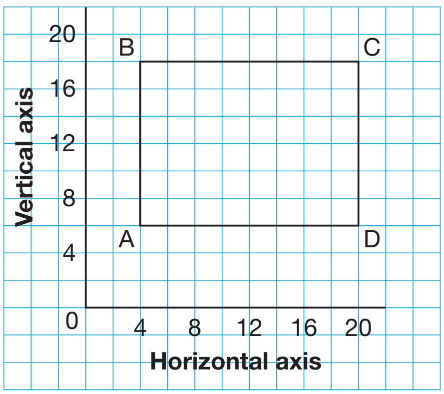
Connect

We can use ordered pairs to describe the position of a shape on a Cartesian plane.

Recall that the Cartesian plane is often called a coordinate grid.

➤ Aria is designing a rectangular playground for a local park in Victoria.

To help plan the playground, Aria drew a rectangle on a coordinate grid. She used the scale 1 square represents 2 m.



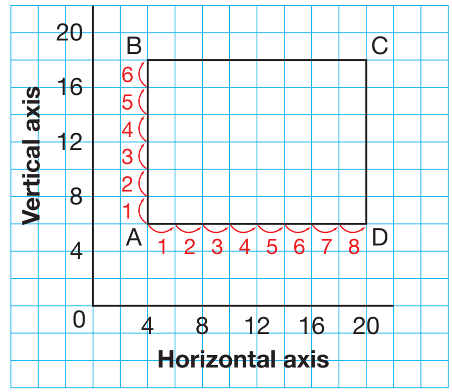
To describe the rectangle, we label its vertices with letters. The letters are written in order as you move around the perimeter of the shape.

We then use coordinates to describe the locations of the vertices.
Point A has coordinates (4, 6).
Point B has coordinates (4, 18).
Point C has coordinates (20, 18).
Point D has coordinates (20, 6).

➤ Here are 2 strategies students used to find the length and width of the playground.

- Gwen counted squares.

  There are 8 squares along the horizontal segment AD.
  The side length of each square represents 2 m.
  So, the playground has length:
  $8 \times 2\text{ m} = 16\text{ m}$

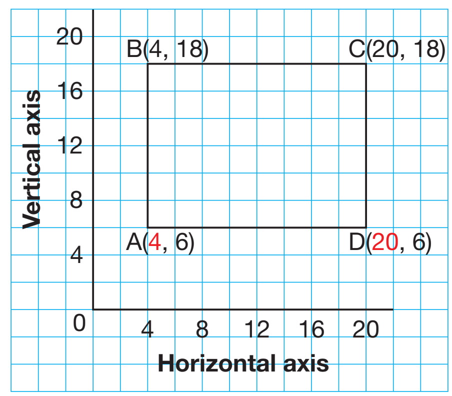


  There are 6 squares along the vertical segment AB.
  The side length of each square represents 2 m.
  So, the playground has width:
  $6 \times 2\text{ m} = 12\text{ m}$

- Jarrod used the coordinates of the points.

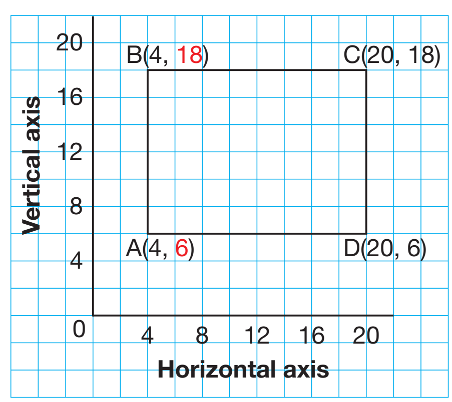
  The first coordinate of an ordered pair tells how far you move right.

  The horizontal distance between D and A is:
  $20 - 4 = 16$
  So, the playground has length 16 m.



  The second coordinate of an ordered pair tells how far you move up.

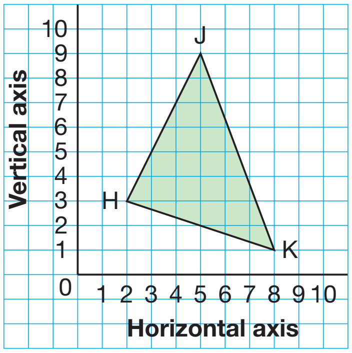
  The vertical distance between B and A is:
  $18 - 6 = 12$
  So, the playground has width 12 m.



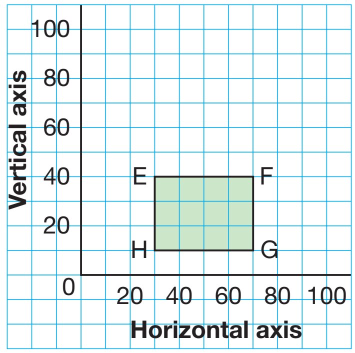
Practice

1. Write the coordinates of the vertices of each shape.

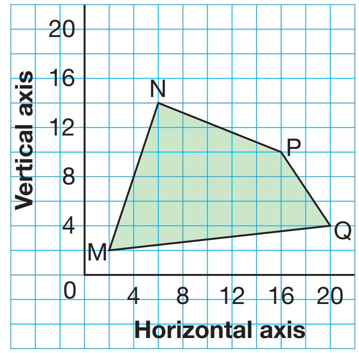
a)



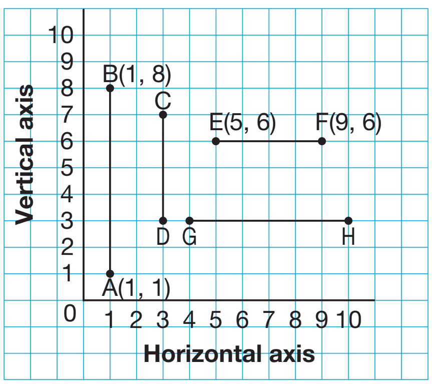
b)



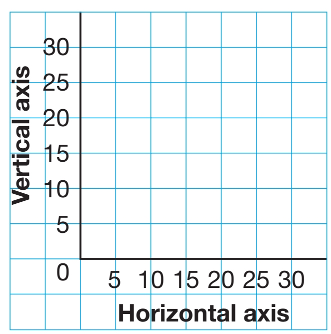
c)



2. Find the length of each line segment on this coordinate grid.
   Describe the strategy you used.



3. Copy this grid.



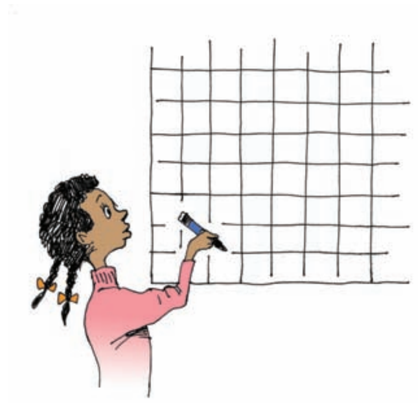
   a) Plot each point on the grid.
      A(10, 5) B(5, 15) C(10, 25)
      D(20, 25) E(25, 15) F(20, 5)
   b) Join the points in order. Then join F to A.
   c) Describe the shape you have drawn.

4. Draw and label a coordinate grid.
   a) Plot each point on the grid.
      What scale will you use? Explain your choice.
      J(4, 2) K(4, 10) L(10, 12) M(10, 4)
   b) Join the points in order. Then join M to J.
      Describe the shape you have drawn.

5. Draw a shape on a coordinate grid.
   Each vertex should be at a point where grid lines meet.
   List the vertices of the shape, in order.
   Trade lists with a classmate. Use the list to draw your classmate's shape.

6. Draw and label a coordinate grid.
   a) Plot each point on the grid.
      What scale will you use?
      Explain your choice.
      A(10, 30)   B(35, 30)   C(35, 15)   D(10, 15)
   b) Join the points in order. Then join D to A.
      Describe the shape you have drawn.
   c) Find the length of each side of the shape.
      Show your work.

7. Draw and label a coordinate grid.
   a) Plot the points A(5, 1) and B(5, 5).
      Join the points.
   b) Find point C so that △ABC is isosceles.
      How many different ways can you do this?
      Draw each way you find.
      Write the coordinates of C.
      How do you know each triangle is isosceles?
   c) Find point D so that △ABD is scalene.
      Show 3 different scalene triangles.
      Write the coordinates of D.
      How do you know each triangle is scalene?



8. Draw and label a coordinate grid.
   a) Plot these points: E(5, 1), F(3, 3), G(5, 6)
   b) Find the coordinates of Point H that forms Kite EFGH.
      Explain the strategy you used.

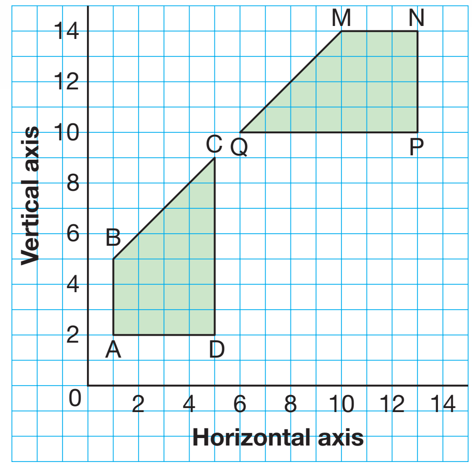
9. The points A(10, 8) and B(16, 8) are two vertices of a square.
   Plot these points on a coordinate grid.
   a) What are the coordinates of the other two vertices?
      Find as many different answers as you can.
   b) What is the side length of each square you drew?

Reflect

How do you decide which scale to use when plotting a set of points on a grid?
Is more than one scale sometimes possible? Explain.

LESSON 2

Transformations on a Coordinate Grid



Translations, rotations, and reflections are transformations.

- Which transformation moves Quadrilateral ABCD to its image, Quadrilateral NMQP?
- What are the coordinates of the vertices of the quadrilateral and its image?

Explore

Game

You will need:

- scissors
- Shape Cards
- coordinate grids
- tracing paper
- Transformation Cards

It's a Transforming Experience!

➤ Cut out the Transformation Cards and the Shape Cards. Shuffle each set of cards. Place the cards face down in separate piles.

➤ Player A takes one card from each pile. On the grid, Player A:
  – draws and labels the shape described on the Shape Card
  – draws and labels the image of the shape after the transformation described on the Transformation Card

➤ If you are able to draw the image of the shape, you score 2 points. If you are not able to draw the image, you score no points.

➤ Switch roles. Continue to play until each player has had 4 turns. The player with more points wins.

Show *and* Share

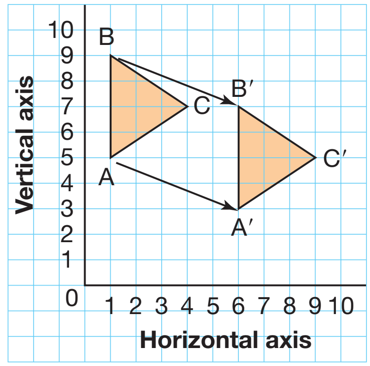
Share your work with another pair of students. What strategies did you use to draw the images?

Lesson Focus | Apply and describe transformations on a coordinate grid.

Connect

Translation

Triangle ABC was translated
5 squares right and 2 squares down.
Its translation image is △A′B′C′.



Vertices of △ABC	Vertices of △A′B′C′
A(1, 5)	A′(6, 3)
B(1, 9)	B′(6, 7)
C(4, 7)	C′(9, 5)

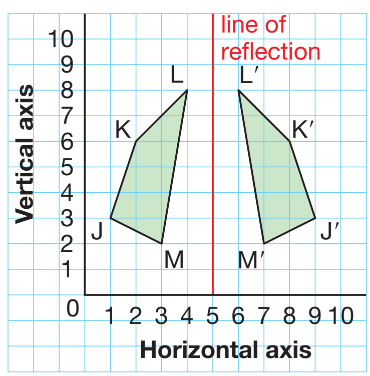
Point A′ is the image of point A.
We write: A′
We say: "A prime"

Each vertex moved 5 squares right and 2 squares down to its image position.

After a translation, a shape and its image face the same way.
The shape and its image are congruent.
That is, corresponding sides and corresponding angles are equal.
We can show this by measuring.

Reflection

Quadrilateral JKLM was reflected in a vertical line through the horizontal axis at 5.
Its reflection image is Quadrilateral J′K′L′M′.



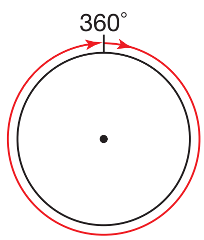
Vertices of Quadrilateral JKLM	Vertices of Quadrilateral J′K′L′M′
J(1, 3)	J′(9, 3)
K(2, 6)	K′(8, 6)
L(4, 8)	L′(6, 8)
M(3, 2)	M′(7, 2)

Each vertex moved horizontally so the distance between the vertex and the line of reflection is equal to the distance between its image and the line of reflection.

After a reflection, a shape and its image face opposite ways.
The shape and its image are congruent.
We can show this by tracing the shape, then flipping the tracing.
The tracing and its image match exactly.

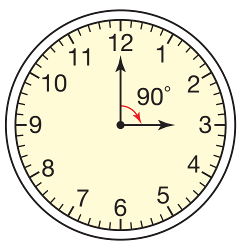
Rotation

When a shape is turned about a point, it is rotated.
A complete turn measures 360°.

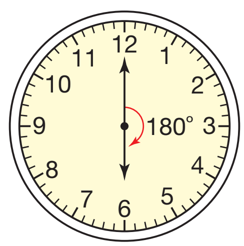


A rotation can be clockwise or counterclockwise.

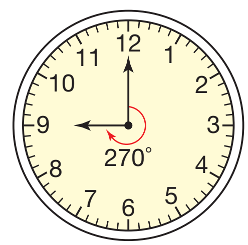
So, we can name fractions of turns in degrees.



A $\frac{1}{4}$ turn is a 90° rotation.

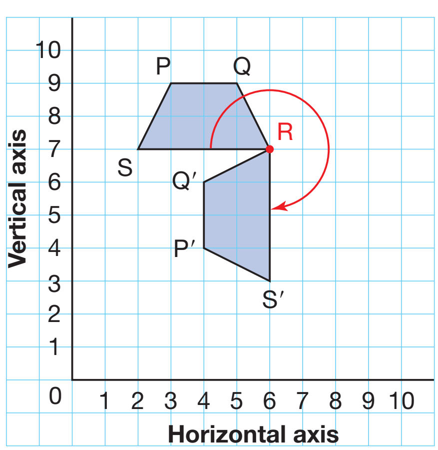


A $\frac{1}{2}$ turn is a 180° rotation.



A $\frac{3}{4}$ turn is a 270° rotation.

Trapezoid PQRS was rotated a $\frac{3}{4}$ turn clockwise about vertex R.
Its rotation image is Trapezoid P′Q′RS′.

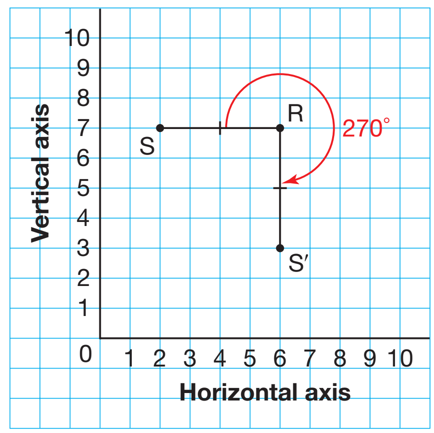


After a $\frac{3}{4}$ turn clockwise, the reflex angle between RS and RS′ is 270°.

Vertices of Trapezoid PQRS	Vertices of Trapezoid P′Q′RS′
P(3, 9)	P′(4, 4)
Q(5, 9)	Q′(4, 6)
R(6, 7)	R(6, 7)
S(2, 7)	S′(6, 3)

The sides and their images are related.
For example,

- The distances of S and S′ from the point of rotation, R, are equal; that is, SR = RS′.
- Reflex ∠SRS′ = 270°, which is the angle of rotation.



After a rotation, a shape and its image may face different ways.
Since we trace the shape and use the tracing to get the image, the shape and its image are congruent.

Practice

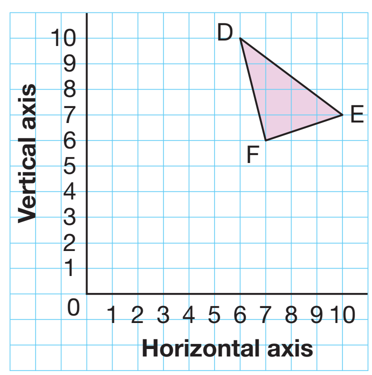
Use tracing paper or a Mira when it helps.

1. Copy this triangle on a grid.

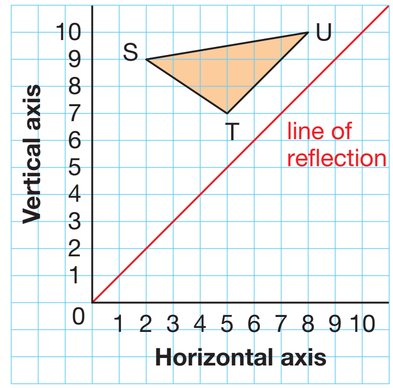
a) Draw the image of △DEF after the translation 6 squares left and 1 square down.

b) Write the coordinates of the vertices of the triangle and its image.
How are the coordinates related?

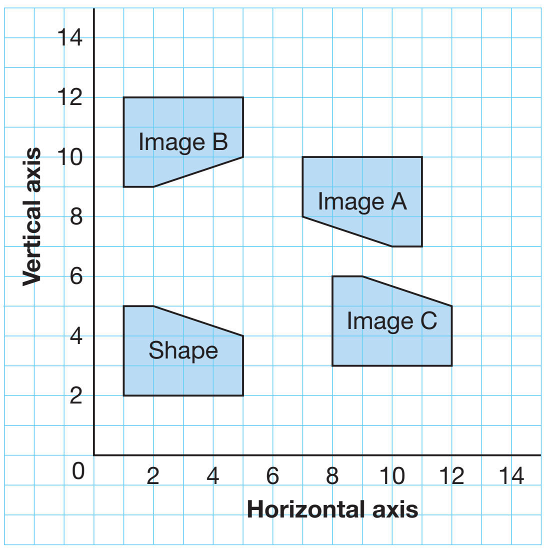
c) Another point on this grid is G(10, 2).
Use your answer to part b to predict the coordinates of point G′ after the same translation.



2. Copy this triangle on a coordinate grid.
   a) Draw the image of △STU after a reflection in the line of reflection.
   b) Write the coordinates of the vertices of the triangle and its image. Describe how the positions of the vertices of the shape have changed.
   c) Another point on this grid is V(4, 3). Predict the location of point V′ after a reflection in the same line. How did you make your prediction?

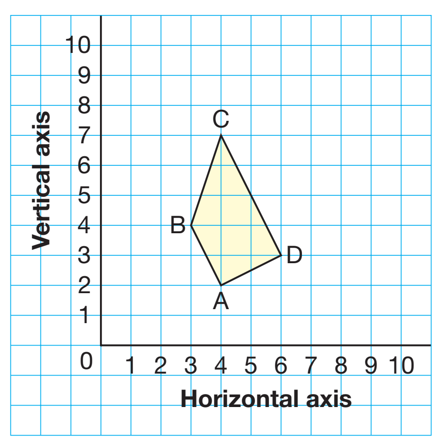


3. This diagram shows a shape and its image after 3 different transformations.

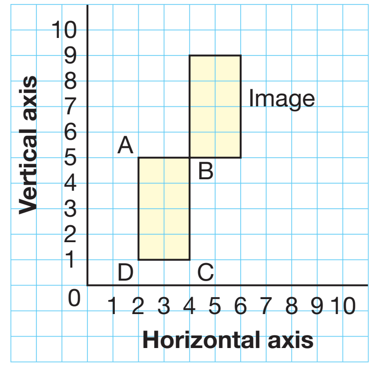


Identify each transformation.
Explain how you know.
   a) the shape to Image A
   b) the shape to Image B
   c) the shape to Image C

4. Copy this quadrilateral on a coordinate grid. Trace the quadrilateral on tracing paper. Draw the image of the quadrilateral after each rotation below. Write the coordinates of the vertices.
   a) 90° clockwise about vertex B
   b) 270° clockwise about vertex B
   c) 270° counterclockwise about vertex B



5. Copy the rectangle and its image on a coordinate grid.
   a) Describe as many different transformations as you can that move the rectangle to its image.
   b) For each transformation:
      - Label the vertices of the image.
      - Describe how the positions of the vertices of the rectangle have changed.

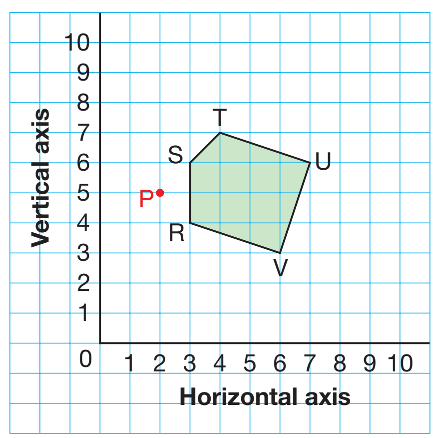


6. A quadrilateral has these vertices:
   Q(5, 2), R(4, 5), S(9, 4), T(6, 3)
   Draw the quadrilateral on a coordinate grid.
   For each transformation below:
   - Draw the image.
   - Write the coordinates of the vertices of the image.
   - Describe how the positions of the vertices of the quadrilateral have changed.

   a) a translation of 3 squares left and 1 square down
   b) a rotation of 90° clockwise about vertex S
   c) a reflection in the horizontal line through the vertical axis at 6

7. Copy this pentagon on a coordinate grid.
   Write the coordinates of each vertex.
   For each transformation below:
   - Draw the image.
   - Write the coordinates of the vertices of the image.
   - Describe how the positions of the vertices of the pentagon have changed.

   a) a translation 2 units right and 3 units up
   b) a reflection in the vertical line through the horizontal axis at 5
   c) a rotation of 90° counterclockwise about P

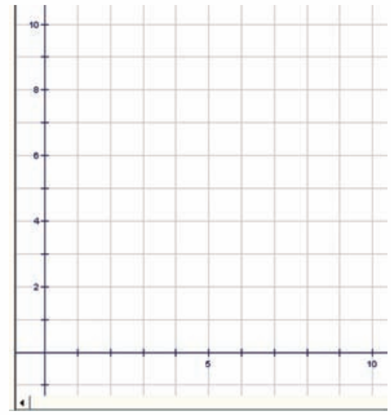


Reflect

How does a coordinate grid help you describe a transformation of a shape?

Using Technology to Perform Transformations

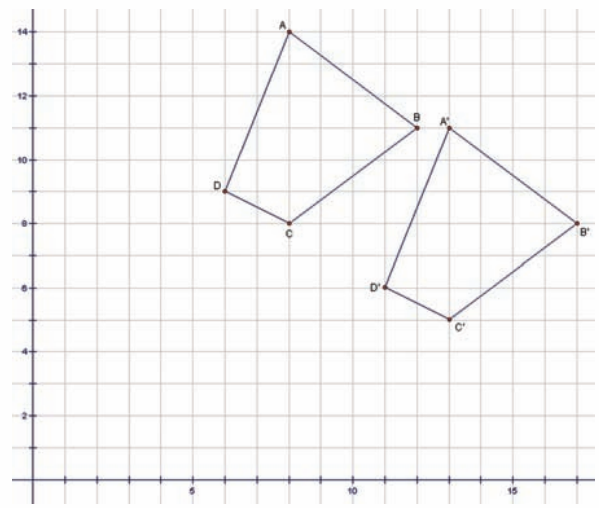
We can use geometry software to transform shapes.



Use dynamic geometry software.
Open a new sketch.
Display a coordinate grid.
Move the origin to the bottom left of the screen.
Check that the distance units are centimetres.
If you need help at any time, use the Help menu.

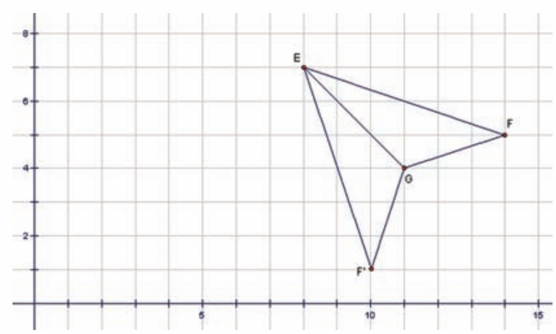
Translating a Shape

- Construct Quadrilateral ABCD.
  Record the coordinates of each vertex.
- Select the quadrilateral.
- Translate the quadrilateral 5 squares right and 3 squares down.
- Label the vertices.
- Write the coordinates of the vertices of the translation image.
- Print the quadrilateral and its image.



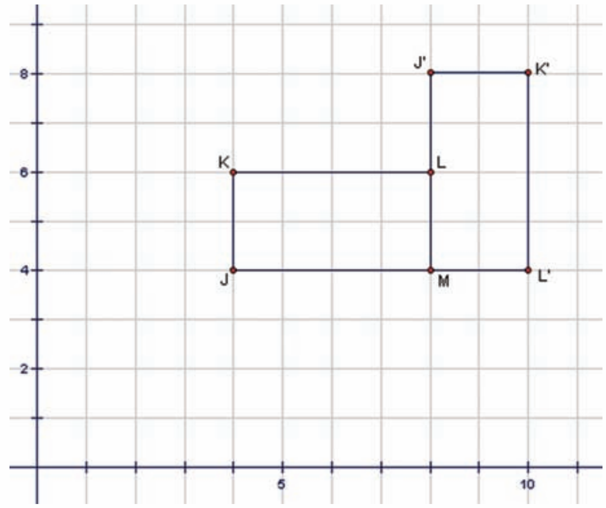
Reflecting a Shape

- Construct $\triangle$EFG.
  Record the coordinates of each vertex.
- Select one side of the triangle as the line of reflection.
- Select the triangle.
- Reflect it in the line of reflection.
- Label the vertices.
- Write the coordinates of the vertices of the reflection image.
- Print the triangle and its image.



Rotating a Shape

- Construct Rectangle JKLM.
  Record the coordinates of each vertex.
- Select a vertex of the rectangle as the point of rotation.
- Select the rectangle.
- Rotate it 270° counterclockwise.
- Label the vertices.
- Write the coordinates of the vertices of the rotation image.
- Print the rectangle and its image.



1. Construct a different shape.
   Label its vertices.
   Record the coordinates of each vertex.
   a) Choose a translation.
   Translate the shape.
   b) Choose a reflection.
   Reflect the shape.
   c) Choose a rotation.
   Rotate the shape.

   For each transformation image:
   - Label, then write the coordinates of the vertices.
   - Describe how the positions of the vertices of the shape have changed.
   - Print your work each time.

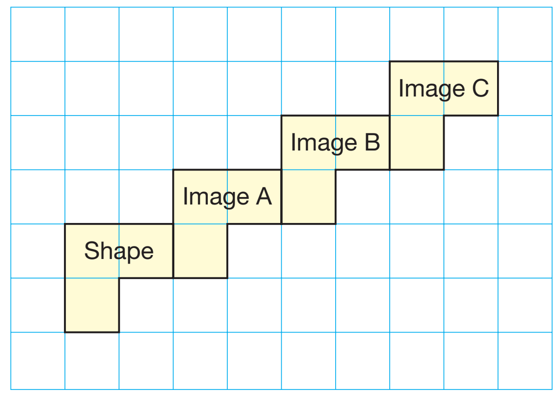
Reflect

Do you prefer to transform a shape using geometry software or using paper and pencil? Explain your choice.

LESSON 3

Successive Transformations

Which type of transformation does this diagram show?
Describe a transformation that moves the shape directly to Image C.



Explore

Game

You will need an 11 by 11 geoboard, 3 colours of geobands, a Mira, tracing paper, and grid paper.

Transformation Challenge

- Player 1 uses a geoband to make a shape.
  Player 2 names a transformation.
  Player 1 uses the transformation to make Image A.
  With Image A as the shape, he then uses the same transformation to make Image B.
  If the transformation cannot be done twice, Player 2 loses 1 point.
- Player 1 draws the shape and its images on grid paper.
  He then names a single transformation that would move the shape directly to Image B.
  Player 1 scores 1 point for each correct transformation he names.
  Player 2 uses the geoboard to check.
- Players switch roles and repeat.
- The first player to get 10 points wins.

Lesson Focus | Draw and describe the image of a shape after repeated transformations.

Show *and* Share

Share your transformations with another pair of students.
What strategies did you use to identify the single transformations?
What do you know about a shape and each of its images?
How can you show this?

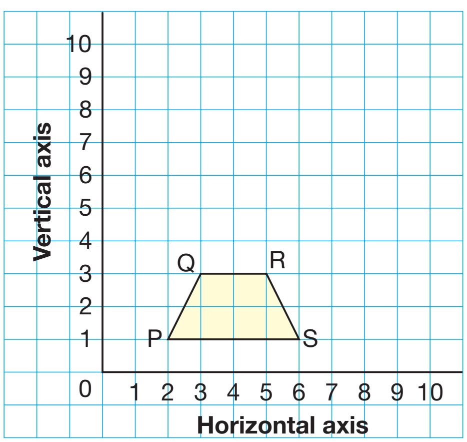
Connect

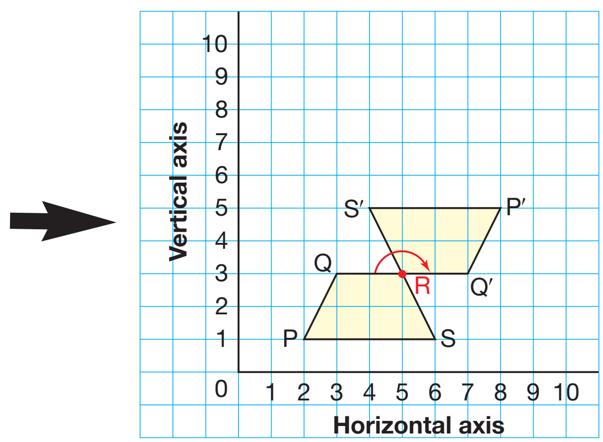
The same transformation can be applied to a shape more than once.

➤ When a shape is translated two or more times, we say the shape undergoes **successive translations**. The same translation may be repeated, as shown at the top of page 303, or the translations may be different.

The same is true for rotations and reflections.

➤ Trapezoid PQRS undergoes **successive rotations**:
- It is rotated 180° about vertex R.
- Then, its image is rotated 90° clockwise about its top right vertex.



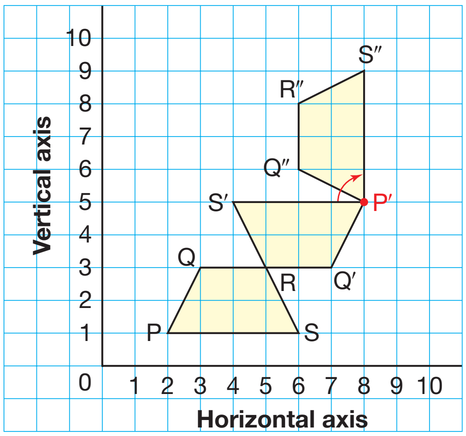


To find the image after the first rotation:

- Trace Trapezoid PQRS on tracing paper.
- Rotate the tracing 180° about R.
- Mark the positions of the vertices of the image.
- Draw the rotation image.
- Label the vertices P′Q′RS′.

To find the final image:

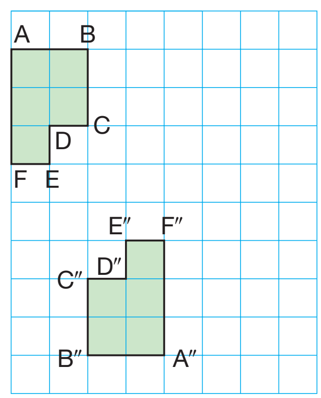
- Trace Trapezoid P′Q′RS′.
- Rotate the tracing 90° clockwise about its top right vertex, P′.
- Mark the positions of the vertices of the image.
- Draw the rotation image.
- Label the vertices P′Q″R″S″.



Read Q″ as "Q double prime."

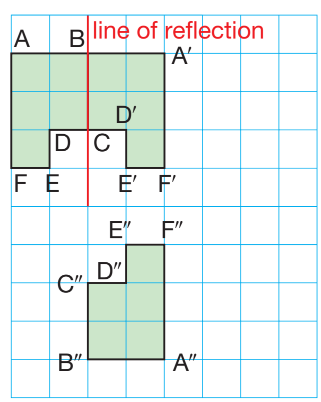
The trapezoid and both its images are congruent.
That is, corresponding sides and corresponding angles are equal.
We know this because we traced the trapezoid each time.

➤ Hexagon A″B″C″D″E″F″ is the image of Hexagon ABCDEF after two **successive reflections**.



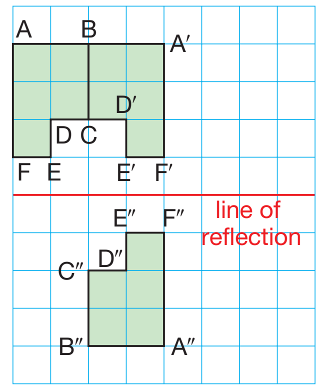
To identify the reflections:

- Reflect the original hexagon so that the image of AF is on the same grid line as A″F″.
  The line of reflection passes through side BC.
- Draw the reflection image of Hexagon ABCDEF.
  This is Image A′BCD′E′F′.



You might need to use guess and test or a Mira to find the lines of reflection.

A′BCD′E′F′ and A″B″C″D″E″F″ face opposite ways and are equal distances from the horizontal line halfway between E′F′ and E″F″.
So, this is the line of reflection.

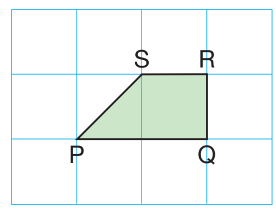


Hexagon A″B″C″D″E″F″ is the image of Hexagon ABCDEF after a reflection in the line through BC, followed by a reflection in the horizontal line halfway between E′F′ and E″F″.

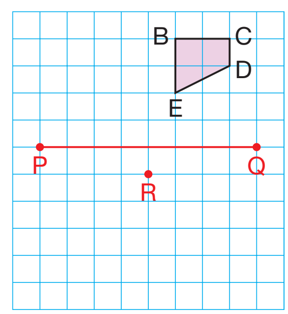
If we trace the hexagon and superimpose it on each image, we see that they match exactly.
The original hexagon and both its images are congruent.

Practice

You will need grid paper, tracing paper, and a Mira.

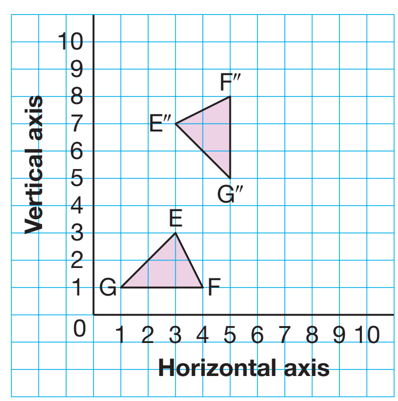


1. Copy this quadrilateral on grid paper. Make:
- **a)** 3 successive translations of 1 square right and 2 squares up
- **b)** 3 successive reflections in the line through SR
- **c)** 3 successive rotations of 180° about vertex R



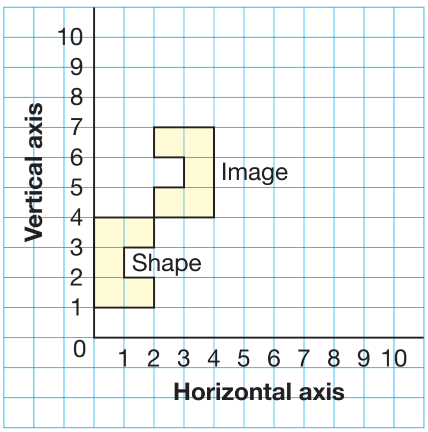
2. Copy this diagram on grid paper.
Draw and label both images each time.
- **a)** Translate the quadrilateral 3 squares left and 2 squares down. Then translate the image 1 square right and 3 squares down.
- **b)** Reflect the quadrilateral in a line through BE. Then reflect the image in the line PQ.
- **c)** Rotate the quadrilateral 90° counterclockwise about vertex E. Then rotate the image 180° about point R.

3. Describe two successive transformations that move △EFG to its image, △E″F″G″.
Show your work.



4. Draw a triangle on grid paper.
   a) Choose two successive translations, reflections, or rotations.
      Apply the first transformation to the triangle.
      Then apply the second transformation to the image.
   b) Label the vertices of each image.
   c) What can you say about the triangle and the images?
      How could you check this?
   d) Describe a single transformation that would move the triangle directly to its final image.

5. a) Describe two successive transformations that move the octagon to its image.



   b) Can you find two other successive transformations? Explain.

6. The coordinates of a shape are:

A(3, 2)	B(3, 6)	C(5, 6)
D(6, 4)	E(5, 3)	F(5, 2)

   - The shape is translated 3 squares right and 1 square up.
   - Then, the image is translated 2 squares left and 2 squares up.
   - Then, the image is translated 1 square left and 3 squares down.

   What are the coordinates of the final image?
   How have the positions of the vertices of the shape changed? Explain.

Reflect

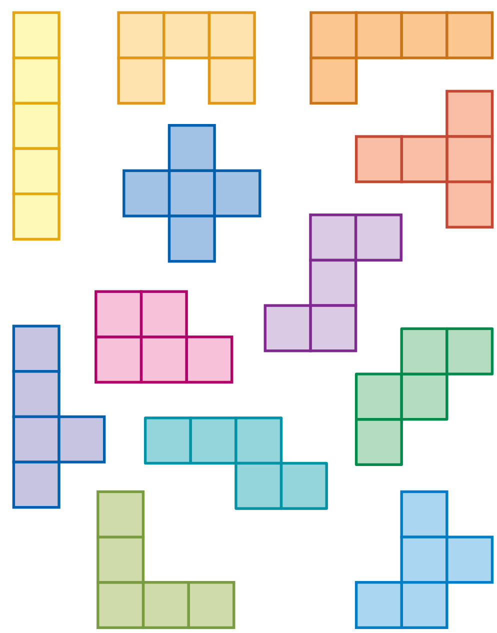
Give a real-world example of successive:
- translations
- reflections
- rotations

LESSON 4

Combining Transformations

Explore — Game

You will need grid paper, scissors, and tracing paper.
Your teacher will give you a large copy of these pentominoes.
Cut out the pentominoes.



What's My Move?

- Each of you chooses 1 pentomino.
  Draw or trace your pentomino on the grid paper.
  Trade grids and pentominoes with your partner.
- Select and record 2 different transformations.
  Keep the transformations secret from your partner.
  Apply one transformation to your partner's pentomino.
  Then apply the second transformation to the image.
  Draw only the second image.
  Return the grid to your partner.
- Identify the combined transformations that moved the pentomino to the final image.
  You score 1 point if you identify the transformations correctly.
- Repeat the game as many times as you can.
  The person with more points wins.

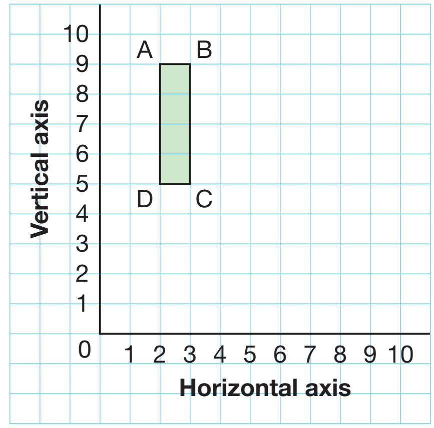
Show *and* Share

Share your transformations with another pair of students.
What strategies did you use to identify your partner's transformations?
In each case, are the pentomino and each of its images congruent?
How can you tell?

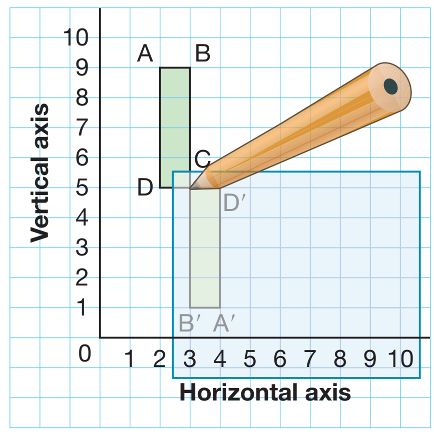
Connect

A combination of 2 or 3 different types of transformations can be applied to a shape.

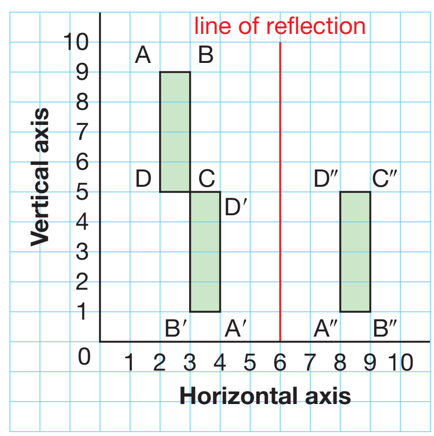
➤ To find the final image of Rectangle ABCD after a rotation of 180° about C, followed by a reflection in a vertical line through 6 on the horizontal axis:



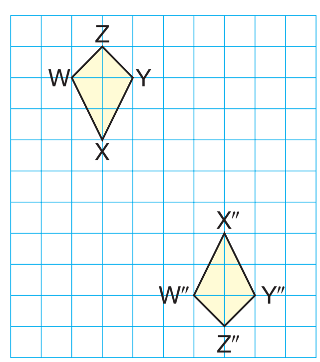
Trace Rectangle ABCD on tracing paper.
Rotate the tracing 180° about C.
Mark the positions of the vertices of the rotation image.
Draw the rotation image.
Label the vertices A′B′CD′.



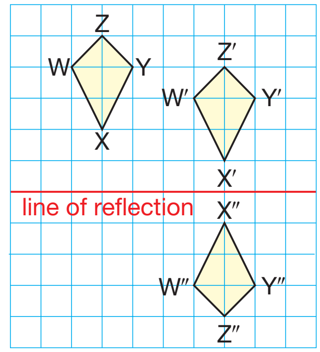
Draw the line of reflection through 6 on the horizontal axis. Reflect the rotation image in the line of reflection.
Each vertex of the reflection image is the same distance from the line of reflection as the corresponding vertex on the rotation image.
The shape and both images are congruent.



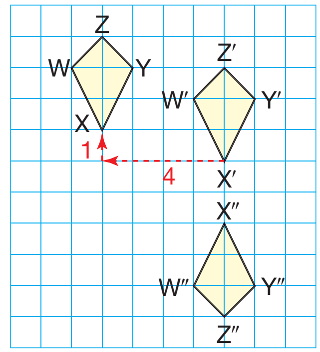
➤ Kite W″X″Y″Z″ is the image of Kite WXYZ after two transformations.



To identify the transformations:
Work backward.
Kites WXYZ and W″X″Y″Z″ face opposite ways.
This suggests a reflection.
A possible line of reflection is the horizontal line 1 square above X″.
Draw the reflection image of Kite W″X″Y″Z″.
This is Kite W′X′Y′Z′.



Kites WXYZ and W′X′Y′Z′ face the same way.
This suggests a translation.
To go from X′ to X, move 4 squares left and 1 square up.

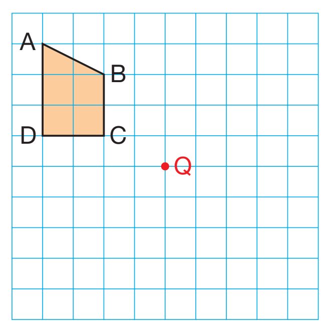


So, to move Kite WXYZ to Kite W″X″Y″Z″ we translate 4 squares *right* and 1 square *down*, then reflect in the horizontal line 1 square *below* X′.

This is one combination of transformations that moves the shape to its final image.
Often, more than one combination is possible.

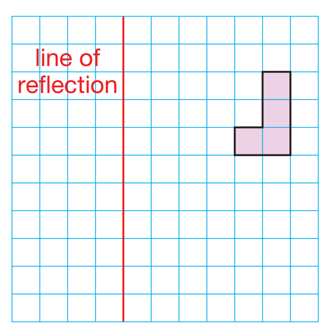
Practice

You will need grid paper, tracing paper, and a Mira.



1. **a)** Copy the quadrilateral on grid paper.
- Translate the quadrilateral 3 squares right.
- Then rotate the translation image 90° clockwise about point Q.

b) Draw and label both images.
c) What can you say about the quadrilateral and its final image? How can you check?



2. **a)** Copy the hexagon on grid paper.
- Translate the hexagon 2 squares left and 3 squares down.
- Then reflect the translation image in the line of reflection.

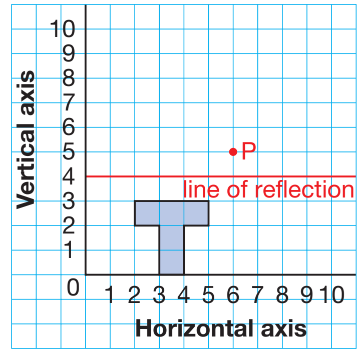
b) Draw and label both images.
c) How can you check that the hexagon and both images are congruent?

3. a) Copy the octagon on a coordinate grid.
   - Reflect the octagon in the line of reflection.
   - Then rotate the reflection image 270° counterclockwise about P.

   b) Draw and label both images.

   c) What are the coordinates of the vertices of the final image?

   d) Are the octagon and its final image congruent? How do you know?



4. Draw and label a quadrilateral on grid paper.

   a) Choose two different transformations.
   - Apply the first transformation to the quadrilateral.
   - Then apply the second transformation to the image.

   What can you say about the quadrilateral and its images? How can you check?

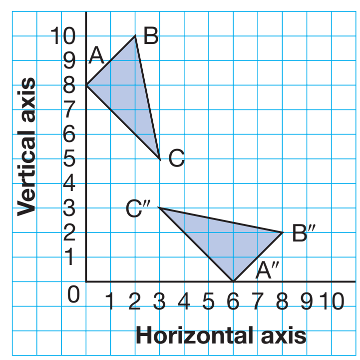
   b) Use a different colour.
   Apply the transformations from part a in the reverse order.

   c) Compare the final images from parts a and b.
   Does the order in which transformations are applied matter? Explain.

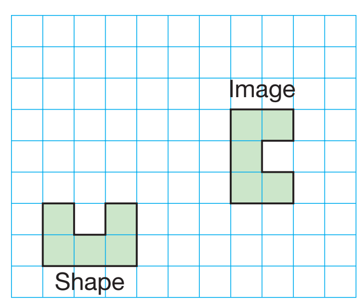
5. Triangle A″B″C″ is the image of △ABC after 2 transformations.

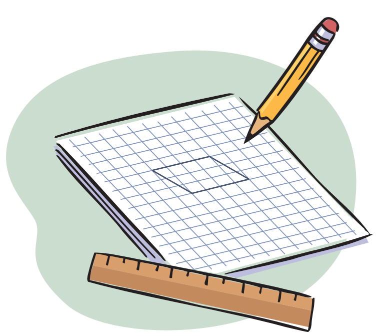
   a) Describe a pair of transformations that move the triangle to its final image.
   Show your work.

   b) Can you find another pair of transformations?
   If your answer is yes, describe the transformations.
   If your answer is no, explain why not.



6. Describe a pair of transformations that move the shape to its image.
Find as many pairs of transformations as you can.





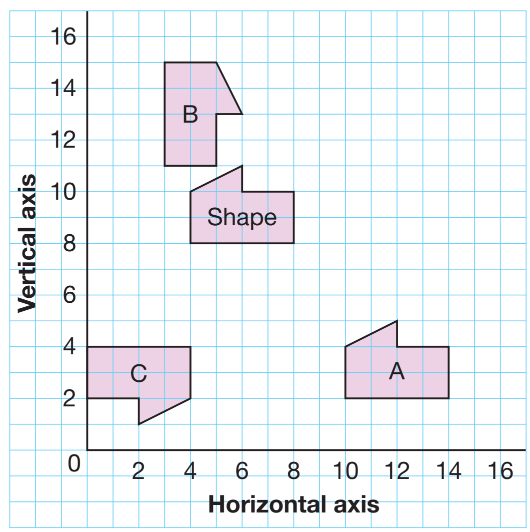
Math Link

First Nations Art

Many First Nations artists use beads and braiding in their work. They produce many items, including jewellery, belts, purses, moccasins, and mukluks. We can often see transformations in the designs used by these artists.
What transformations do you see in the beading on these mukluks?

7. The coordinates of the vertices of a pentagon are:
   A(7, 3) B(6, 4) C(6, 5) D(7, 6) E(8, 5)
   The pentagon is translated 5 squares left and 3 squares up.
   Then, it is reflected in a horizontal line through (0, 5) and (10, 5).
   Then, it is translated 2 squares right and 2 squares up.
   a) What are the coordinates of the final image?
   b) What do you notice about the pentagon and its final image?

8. Describe a pair of transformations that move the shape to each image.
   Can you find more than one pair of transformations for each image?
   Explain.
   a) Image A
   b) Image B
   c) Image C



Reflect

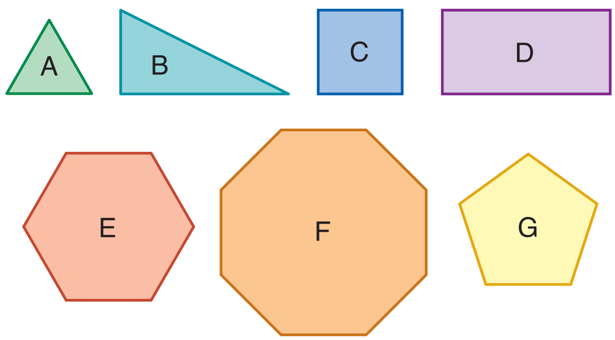
Suppose you know the location of a shape and its final image after 2 transformations.
What strategies can you use to identify the transformations?

LESSON 5

Creating Designs

Explore

You will need tracing paper and scissors.
Your teacher will give you a large copy of these shapes.



Cut out the shapes.

➤ Choose one shape. Make sure it is different from the shapes chosen by others in your group.
Trace copies of the shape to make a design.
Think about translations, rotations, and reflections.
Colour your design.
Write to explain how your design can be created by repeatedly transforming the shape.

➤ Repeat the activity.
This time, try to make a design using 2 different shapes.
Write to explain how your design can be created by repeatedly transforming the 2 shapes.

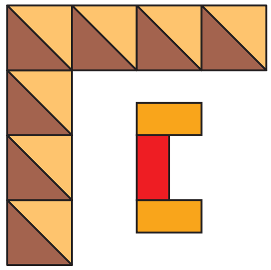
Show *and* Share

Compare your designs with those of a classmate who used the same shapes.
Did you use the same types of transformations? Explain.
Do your designs look the same? Why or why not?

Connect

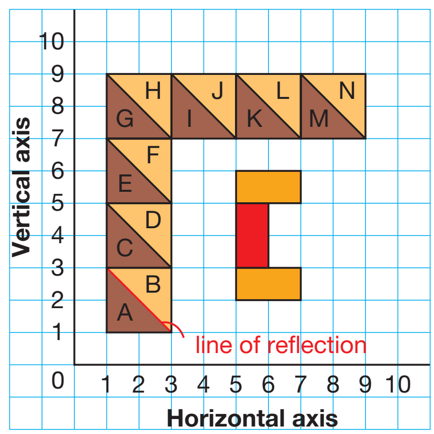
We can use transformations of one or more shapes to create a design.

➤ Calum designed this logo for his local cycling club in Comox Valley, BC.



When creating the logo, Calum worked on a coordinate grid.

There are many transformations in his design.
One possible set of transformations used to create the design is:



Start with Triangle A.
Reflect the triangle in its sloping side to get Image B.
Translate Triangle A two squares up to get Image C.
Reflect Image C in its sloping side to get Image D.
Continue to translate and reflect in this way
to get Images E, F, G, and H.
Or, translate C and D together 2 squares up, twice.

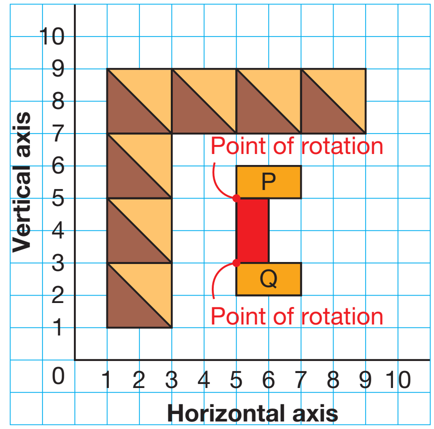
Translate Image G two squares right to get Image I.
Reflect Image I in its sloping side to get Image J.
Continue to translate and reflect in this way to get
Images K, L, M, and N.
Or, translate G and H together 2 squares right, 3 times.

To create the letter C:

Start with the red rectangle.
Rotate the rectangle 90° counterclockwise about point (5, 5) to get Image P.

Rotate the red rectangle 90° clockwise about point (5, 3) to get Image Q.

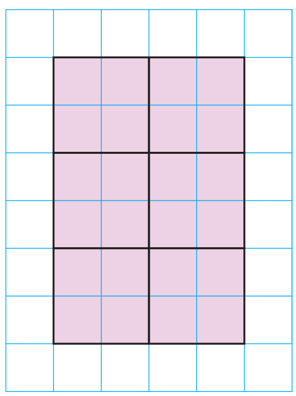
Calum may have used other possible sets of transformations to create his design.



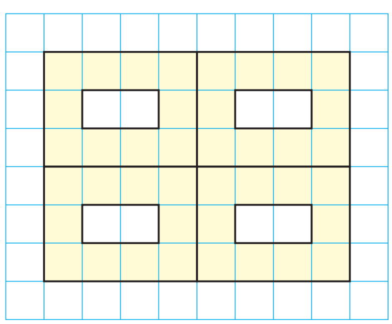
Practice

1. Explain how you could use transformations to make each design.

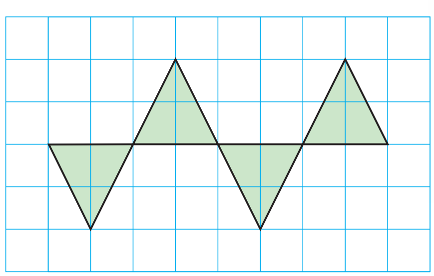
a)

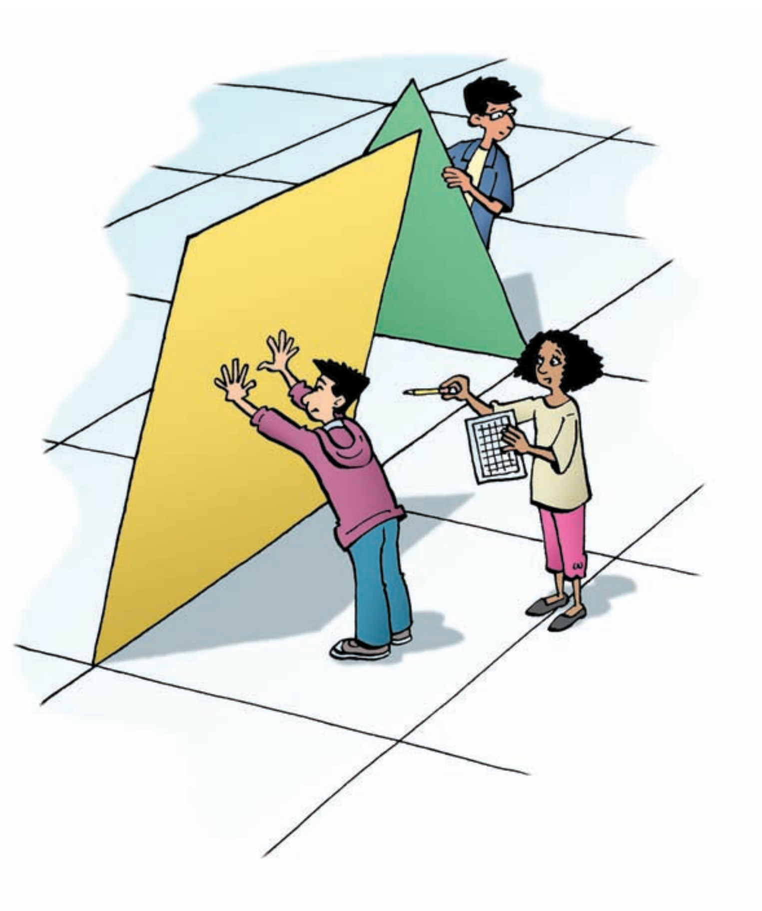


b)



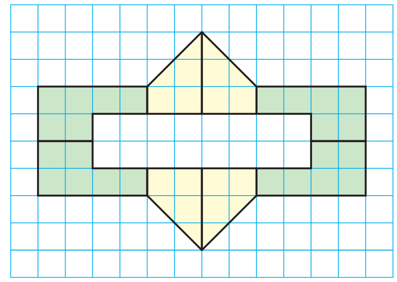
c)





2. Draw a shape on grid paper.
Transform copies of the shape to create a design.
Describe the transformations you used.

3. Recreate this design.
Identify the original shapes.
Describe a set of transformations that could be used to create the design.



4. **a)** Plot these points on a coordinate grid.

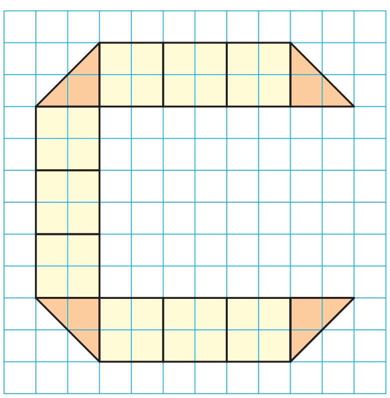
A(2, 0)	B(2, 2)	C(0, 2)	D(0, 4)
E(2, 4)	F(2, 6)	G(4, 6)	H(4, 4)
I(6, 4)	J(6, 2)	K(4, 2)	L(4, 0)

Join the vertices in order. Then join L to A.

b) Translate the shape different ways to make a design.
Describe the translations you used.

c) Use a different transformation to make a design.
Describe the transformations you used.

5. Wahaba designed this logo for her canoe club's trip to Bowron Lake Provincial Park.



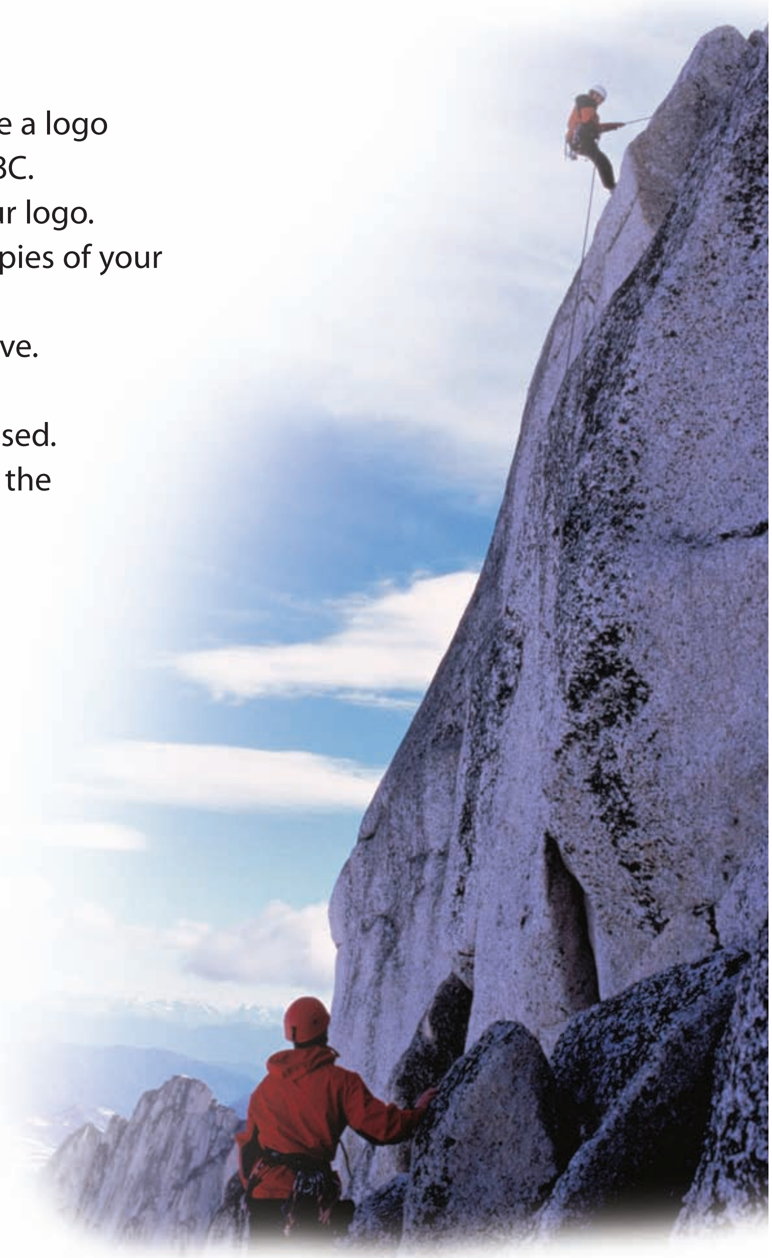
She transformed copies of 2 shapes to create the letter C. The letter C looks like it is made from 3 overlapping canoe-like shapes.

a) What were the original shapes?

b) Describe the transformations that could have been used to create the logo.

c) Is another set of transformations possible?
If your answer is yes, describe the transformations.

6. Suppose you have been hired to create a logo for a rock-climbing club in Squamish, BC.
   a) Choose two or more shapes for your logo.
      Create the logo by transforming copies of your shapes on grid paper.
      Colour your logo to make it attractive.
   b) Identify the original shapes.
      Describe the transformations you used.
   c) Describe how your logo represents the rock-climbing club.



7. This is the Bear Paw quilt block.

   a) Draw a coordinate grid.
      Label the axes from 0 to 7.
   b) Copy the quilt block onto the grid.
   c) The block can be made by transforming shapes.
      - Identify the original shapes.
      - Describe a set of transformations that can be used to create the block.

Reflect

When you see a design with congruent shapes, how do you decide which transformations could have been used to create it?
Use an example to explain.

At Home

Look for designs at home that can be described using transformations. Copy each design. Share the designs with your classmates. Describe a possible set of transformations for each design.

ASSESSMENT FOCUS | Question 6

LESSON 6

Strategies Toolkit

Explore

You will need Pattern Blocks and a Mira.

Choose 3 Pattern Blocks, 2 the same and 1 different.
Arrange the 3 blocks to make a shape with exactly 1 line of symmetry.
Each block must touch at least one other block.
Trace the shape.
Draw a dotted line to show the line of symmetry.

Show *and* Share

Describe the strategy you used to solve the problem.
Could you make more than one shape? Explain.

Connect

You will need pentominoes, grid paper, and a Mira.
Choose 2 different pentominoes.
Arrange the pentominoes to create a shape with exactly 1 line of symmetry.
Trace the shape and show the line of symmetry.

Strategies

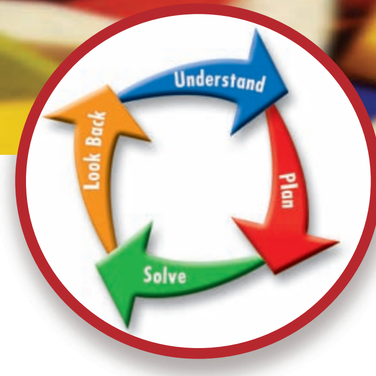
- **Make a table.**
- **Solve a simpler problem.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**

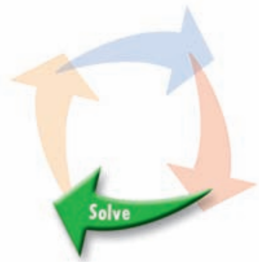
What do you know?

- Use 2 different pentominoes.
- Arrange the pentominoes to make a shape.
- The shape must have exactly 1 line of symmetry.

Think of a strategy to help you solve this problem.

- You can use **guess and test** to find a shape with exactly 1 line of symmetry.





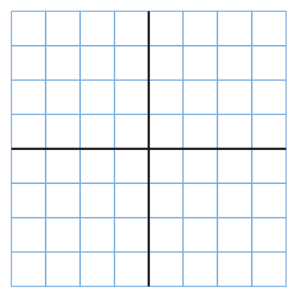
Arrange the pentominoes to make a shape.
Use a Mira to check for lines of symmetry.
If the shape has no lines of symmetry
or more than 1 line of symmetry,
try a different arrangement to make a new shape.

Check your work.
Does your shape have exactly 1 line of symmetry?
How do you know?

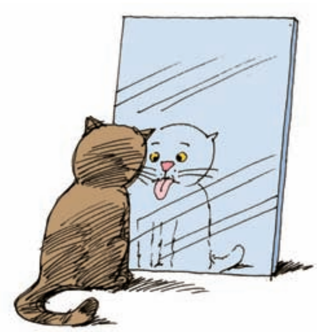
Practice

Choose one of the
Strategies

1. Draw lines of reflection to divide a piece of grid paper into 4 congruent sections.
   a) Draw Shape A in one section.
   Reflect Shape A in one of the lines of reflection.
   Label the image B.
   b) Reflect Image B in the other line of reflection.
   Label the image C.
   c) Describe a transformation that would move Shape A directly to Image C.
   How many different transformations can you find?



2. Repeat question 1.
   This time divide the grid paper into 3 congruent sections.



Reflect

How does guess and test help you solve a problem?
Use pictures and words to explain.

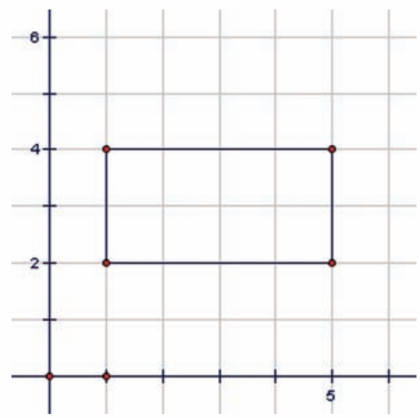
Using a Computer to Make Designs

We can use geometry software and transformations to make designs.

If you need help at any time, use the Help menu.

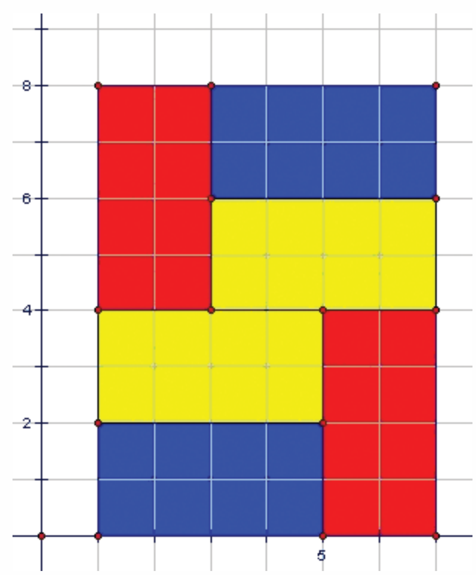
Use dynamic geometry software.
Open a new sketch.
Display a coordinate grid.
Move the origin to the bottom left of the screen.
Check that the distance units are centimetres.

➤ To create a design:
Construct a rectangle.



Use the software to translate, reflect, or rotate the rectangle.

Continue to transform the rectangle or an image rectangle to create a design.
Colour, then print your design.



1. Construct two shapes.
Use transformations to create a design using the two shapes.
Colour your design.
Identify and describe the transformations used to make the design.

Reflect

What are the advantages of using a computer to create a design?
Are there any disadvantages? Explain.

Unscramble the Puzzle

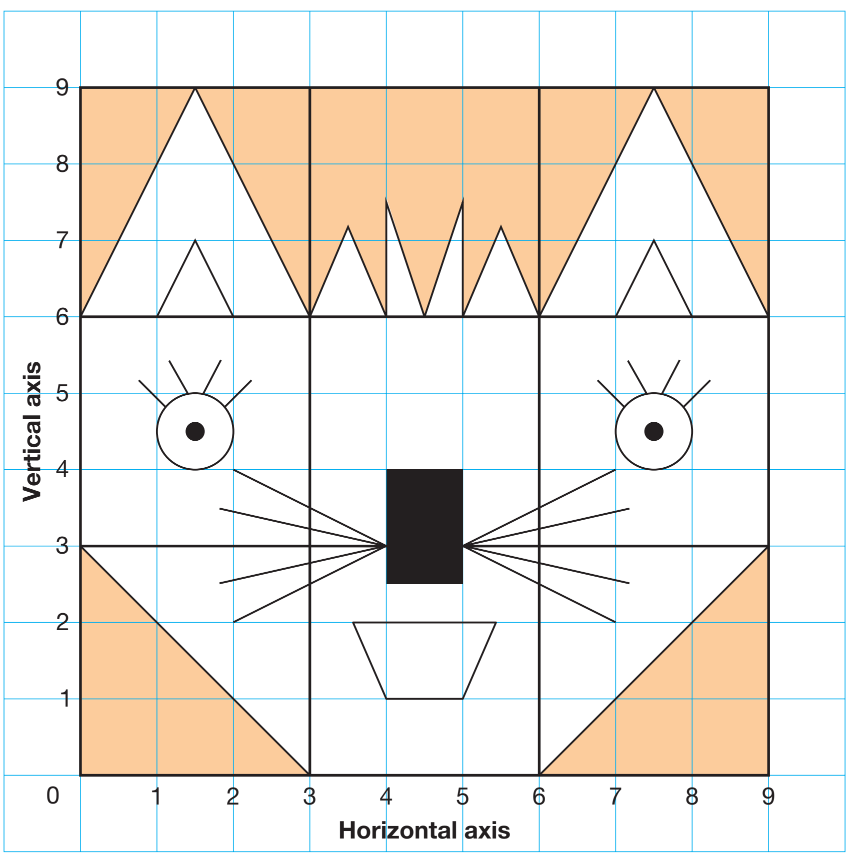
In this game, you use transformations to put a puzzle together.

You will need 1-cm grid paper, scissors, a ruler, and a pencil.
Your teacher will give you a copy of a mixed-up puzzle.

Work with a partner.

➤ Use and describe transformations to move each piece to its correct spot.
After you describe the transformation, cut out the puzzle piece.
Write the transformation on the back of the piece.
Place the piece on the puzzle below.

➤ The game is over when the puzzle is complete.



Unit 8 Show What You Know

LESSON

1 **1.** Draw and label a coordinate grid.

a) Plot each point on the grid.
What scale will you use? Explain your choice.
A(2, 6) B(4, 14) C(12, 14) D(8, 10) E(10, 2)

b) Join the points in order. Then join E to A.
Describe the shape you have drawn.

c) Find the length of the horizontal side of the shape.

Use tracing paper when it helps.

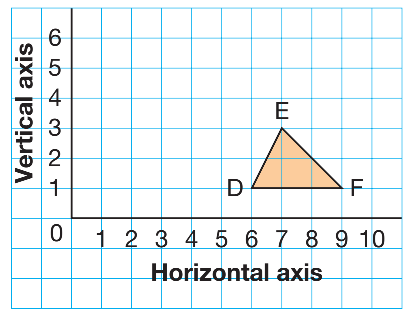
2 **2.** Copy △DEF on a coordinate grid.
For each transformation below:

- Draw the image after the transformation.
- Write the coordinates of the vertices of the image.
- Describe how the positions of the vertices of the triangle have changed.

a) a translation of 4 squares left and 1 square down

b) a reflection in the vertical line through the horizontal axis at 5

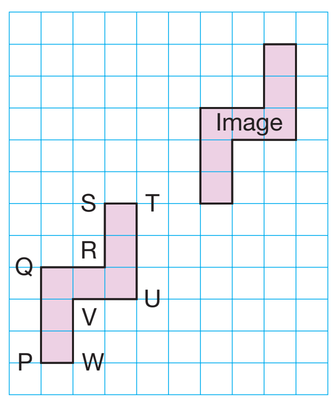
c) a 90° counterclockwise rotation about vertex E



3. Copy octagon PQRSTUVW and its image on grid paper.

a) Describe as many different single transformations as you can that move the octagon to its image.

b) For each transformation, label the vertices of the image.



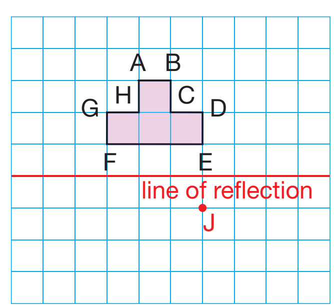
3 **4.** Copy this octagon on grid paper.
Draw and label both images each time.

a) Translate the octagon 2 squares right and 3 squares down.
Then translate the image 4 squares left and 4 squares up.

b) Reflect the octagon in a line through DE.
Then reflect the image in the given line of reflection.

c) Rotate the octagon 90° clockwise about point F.
Then rotate the image 180° about point J.

d) What can you say about the octagon and all its images?



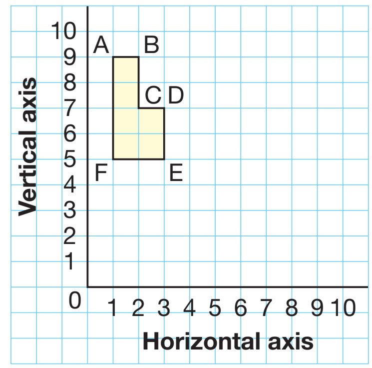
LESSON

4

5. **a)** Copy this hexagon on a coordinate grid.
- Rotate the hexagon 180° about (4, 7).
- Then, reflect the rotation image in a line through FE.

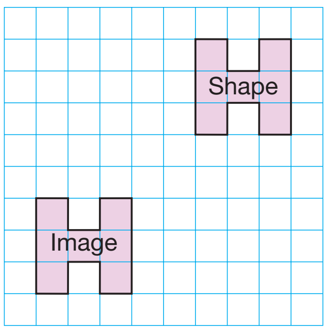
Draw and label both images.

b) What are the coordinates of the vertices of the final image?



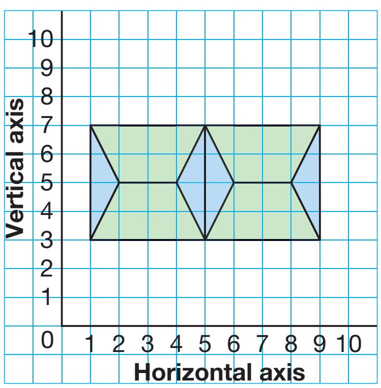
6. **a)** Describe two successive transformations that move the shape to its image.

b) Find as many pairs of transformations as you can.



5

7. This design was formed by repeatedly transforming 2 shapes.



a) Copy the design.
Identify the 2 original shapes.

b) Describe the transformations that could have been used to create the design.

c) Is another set of transformations possible?
If your answer is yes, describe the transformations.

d) Use the 2 original shapes and transformations to make a different design.
Describe the transformations you used.

UNIT 8 Learning Goals

- ✓ draw shapes in the first quadrant of a Cartesian plane
- ✓ draw and describe images on a plane after single transformations
- ✓ draw and describe images after combinations of transformations, with and without technology
- ✓ create a design by transforming one or more shapes
- ✓ identify and describe transformations used to produce an image or a design

Unit Problem

Art and Architecture

Hatley Castle, Victoria, British Columbia

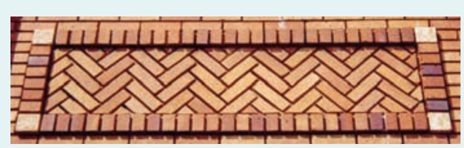
Many buildings have interesting designs that show transformations.

Part 1

These patterns are found on buildings in Saskatchewan.
Identify the transformations in each pattern.

Brick pattern on the Performing Arts Centre in Moose Jaw

Pattern on Bellamy Block in Moose Jaw



Herringbone brick pattern on former Bank of Toronto, in Assiniboia

Check List

Your work should show
- ✓ accurate identification of transformations
- ✓ a building pattern that uses transformations
- ✓ a clear explanation of how you constructed your pattern
- ✓ correct use of geometric language

Part 2

Suppose a new building is to be constructed in your city.
Design a pattern for the outside of the building.
Sketch some shapes you could use in the pattern.
Use the shapes you sketched.
Use transformations to create a pattern.
Colour your pattern.

Part 3

Describe your pattern.
Describe the transformations you used to create your pattern.
Give the building a name.
Where on the building will this pattern be found? Explain.

Reflect on Your Learning

How do you think transformations could be used by an architect, a clothing designer, a bricklayer, or a landscaper?

Investigation

The Domino Effect

You will need dominoes, a metre stick, a stopwatch, and grid paper.

Part 1

➤ Begin with 20 dominoes.
Stand them on end, 3 cm apart. Use a stopwatch.
Push one domino at one end, so all the dominoes fall.
Time how long it takes them to fall.
Record the number of dominoes and the time in a table.

➤ Repeat with 30 dominoes, 40 dominoes, 50 dominoes, up to 80 dominoes.

➤ Describe any patterns you see in the table.

➤ Predict how long it would take 120 dominoes to fall.
How did you make your prediction?

Part 2

Draw a graph to display the data in your table.
Explain your choice of graph.
Describe the graph.
About how long would it take 35 dominoes to topple?
What strategy did you use to find out?

Display Your Work

Report your findings using pictures, numbers, and words.

Take It Further

Investigate different arrangements of dominoes.
What effect does placing the dominoes closer together have on the time it takes them to topple? Explain.
Arrange the dominoes in a curve.
How long does it take them to topple?

Units 1–8 Cumulative Review

Unit

1

1. Mrs. Tetrault wants the students in her Grade 6 class to read each night. She said they should start at 5 min and add 3 min each night until they reach 50 min.
   a) Make a table to show the time spent reading for each of the first 4 nights.
   b) Write a pattern rule that relates the night to the time spent reading.
   c) Write an expression to represent the pattern.
   d) On which night will the students read for 50 min?

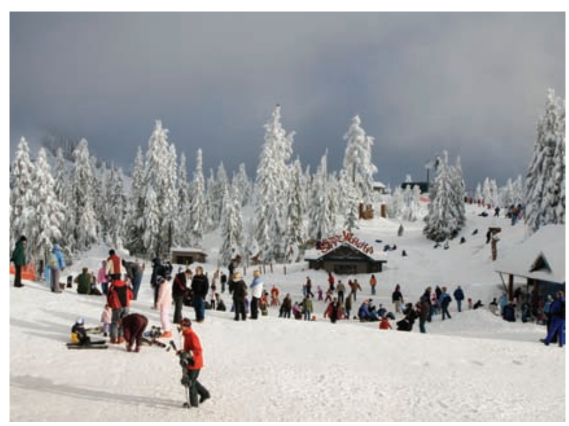
2

2. In the 2006–2007 season, the Western Hockey League had a total attendance of 3 519 007. Write this number in a place-value chart, then in expanded form and in word form.

3

3. Multiply or divide. Which strategies did you use?
   a) 2.737×5
   b) 0.463×3
   c) 14.025×4
   d) $16.488 \div 6$
   e) $\$18.37 \div 3$
   f) $0.133 \div 7$

4. Sidney and his friends save money to go skiing at Grouse Mountain. A daily lift ticket costs $37.00. Sidney saves $5.45 each week for 7 weeks. Does Sidney have enough money to buy a lift ticket? How do you know?



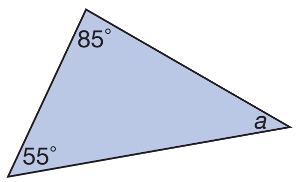
4

5. a) Use a ruler and a protractor. Draw a 35° angle. Which type of angle did you draw?
   b) What is the measure of the outside angle in part a? How do you know? How would you classify this angle?
   c) Use tracing paper to copy the angle in part a. Rotate the angle $\frac{1}{4}$ turn counterclockwise about its vertex. Measure the angle. What do you notice?

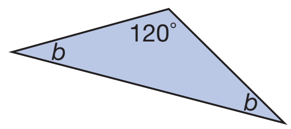
Unit

6. Find the measure of each unknown angle without measuring.

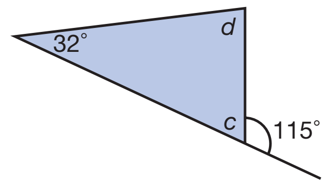
a)



b)



c)



5

7. Write each mixed number as an improper fraction.

a) $2\frac{4}{9}$ **b)** $4\frac{1}{7}$ **c)** $3\frac{3}{8}$ **d)** $1\frac{2}{5}$

8. Write each ratio in as many ways as you can.

a) snowshoes to snowboards
b) snowboards to snowshoes
c) snowboards to snowshoes and snowboards
d) snowshoes to snowshoes and snowboards

9. Write 2 equivalent ratios for each ratio.

a) 5 : 3 **b)** 1 : 6 **c)** 4 : 7 **d)** 1 : 5

6

10. Use a ruler and plain paper to draw 6 different triangles.
Measure each angle.

a) Classify each triangle as acute, right, or obtuse.
Explain how you know.
b) Is any triangle isosceles or equilateral?
How do you know?

11. Bethany sent her pen pal in Baker Lake, Nunavut, a stuffed animal. She packed the stuffed animal into a box that measured 22 cm by 12 cm by 15 cm.
What was the volume of the box?

12. What is your classmates' favourite winter activity?
a) Make a prediction.
b) Design a questionnaire you could use to find out.
c) Ask the question. Tally the results.
d) How did the results compare with your prediction?

13. Would you use a line graph or a series of points to display each set of data? Explain your choices.
a) the height of a corn plant as it grows
b) the life left in a light bulb as it burns
c) the population of your school over the last 10 years

14. This table shows the estimated grizzly bear population on Alberta provincial land (excluding national parks) from 1996 to 2000.
a) Draw a graph to display these data.
b) Explain how you chose the vertical scale.
c) Did you join the points? Explain.
d) What conclusions can you make from the graph?

Year	Estimated Number of Grizzly Bears
1996	765
1997	776
1998	807
1999	833
2000	841

15. Étienne has a collection of foreign coins.
He has 2 coins from Britain, 6 from Japan, 12 from Mexico, and 4 from China.
Assume all the coins have the same size and mass.
Étienne places the coins in a bag and picks one without looking.
a) List the possible outcomes.
b) What is the theoretical probability of each outcome?
- Étienne picks a Chinese coin.
- Étienne picks a Canadian coin.
- Étienne picks a Mexican coin.
- Étienne picks a coin that is not British.

16. Olivie surveyed the Grade 6 students in her school to answer this question:
What do you use the Internet for most often?
The table shows the data she collected.
a) Draw a graph to display these data.
Explain your choice of graph.
b) What do most students use the Internet for?
How does the graph show this?

Use	Number of Students
E-mail	15
Chatting	18
Downloading Music	12
Homework	8
Other	7

Unit
8

17. Draw and label a coordinate grid.

a) Plot each point on the grid.

P(20, 20) Q(20, 60) R(40, 70) S(60, 60) T(50, 10)

b) Join the points in order. Then join T to P.
What scale did you use? Explain your choice.

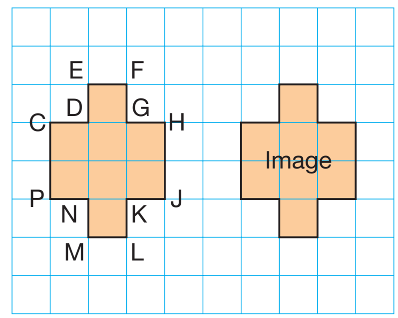
c) Describe the shape you have drawn.

d) Find the length of the vertical side of the shape.

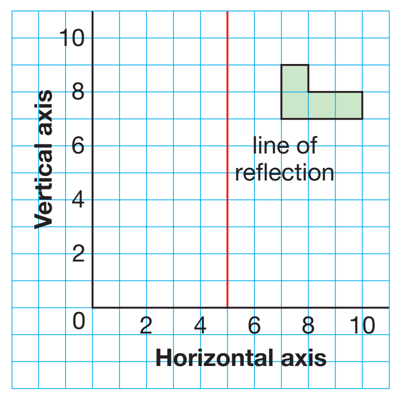
18. Copy this shape and its image on grid paper.

a) Describe as many different single transformations as you can that move the shape to its image.

b) For each transformation, label the vertices of the image.



19. Copy the shape and the line of reflection onto a coordinate grid.
Reflect the shape in the line of reflection.
Then translate the reflection image 5 squares down.
What are the coordinates of the final image?



20. Look at your answer to question 19.
Suppose you translated the shape first, then reflected the translation image in the line of reflection.
What would the coordinates of the final image be?

21. Rhiannon designed this logo for her gardening club in Strathcona, Alberta.
She transformed copies of 2 shapes to make a flower-like shape.

a) Copy the design.
Identify the 2 original shapes.

b) Describe the transformations that could have been used to create the logo.

c) Is another set of transformations possible?
If your answer is yes, describe the transformations.